THE PRINCIPLE BEHIND THE UNCERTAINTY PRINCIPLE

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Whilst physicists have long been aware of the existence of a fundamental uncertainty principle in quantum mechanics, an explicit understanding of this principle has remained an enigma, our grasp limited to specific "uncertainty relations". In this work we overcome these limitations by deconstructing the concepts of "uncertainty" and "joint uncertainty", ultimately extracting the essence of their meaning. We then apply these notions to quantum measurement experiments, resulting in a thorough understanding of the element of uncertainty in quantum mechanics. We identify the essential features of all possible uncertainty relations, which put together constitute our formulation of the Uncertainty Principle. The key idea in our work is that the notions of uncertainty and joint uncertainty can be founded on only the most basic, objectively justifiable axioms. This minimalistic approach leads to the powerful generality of the resulting formulation, motivating us to call it "The" Uncertainty Principle.

In developing our notion of uncertainty, we considered two mechanisms of uncertainty increase: Random symmetry transformations; and classical processing via channels (followed by recovery). Corresponding to these, we identified two classes of doubly stochastic matrices, $\mathcal{D}^{\rm sym}$ and $\mathcal{D}^{\rm rec}$. Uncertainty measures in the strictest sense must be monotonically non-decreasing under both these classes. Surprisingly, we found that the much-used measure of variance can *decrease* under the class $\mathcal{D}^{\rm rec}$, for which reason we classify it as a *weak* uncertainty measure.

Along the way, we show that *the* Uncertainty Principle is not tied to any particular operational execution of the measurements. We will illustrate this by restricting to specific operational frameworks. This will result in universal uncertainty relations for a class of joint uncertainty measures associated with these frameworks that, nevertheless, fall short of capturing all the intricacies of the Uncertainty Principle.

The rich variety of applications of the Uncertainty Principle include cryptographic tasks such as key distribution and information locking. Another practical application is the separability problem in entanglement theory. The continuous-variable version of the Principle plays a role in applications of squeezed states, which are ubiquitous in quantum information processing with continuous variables.

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